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Preface

Special issue on computational harmonic analysis, Part III

In providing a valuable link among engineers, scientists, and mathematicians with a common interest in the applied and computational aspects of harmonic analysis, this interdisciplinary journal also plays a key role in facilitating interactions among researchers who work on similar problem areas. To enhance this important mission and to encourage active participation by newcomers, particularly students and young researchers, a tri-annual international conference was initiated in 2001, with the second meeting held at Vanderbilt University in May, 2004.

This special issue on computational harmonic analysis (SICHA) is an outgrowth of the Vanderbilt Conference organized by the special issue editors. It is intended not to be the proceedings of the conference, but rather to provide the participants an opportunity to submit their research papers, directly or indirectly related to the lectures delivered at the conference, to the journal. All papers published in the SICHA have been reviewed under the same journal guidelines and policy.

This is the third and final part of the SICHA. It consists of eight papers that address various aspects of computational harmonic analysis, particularly in the direction of multi-wavelets and frames, such as introduction of new theories and methods for wavelet characterization and construction, as well as an interesting connection to mobile communication via time–frequency analysis.

The success of the introduction of orthogonal wavelet bases is due to their applicability in many areas of science, engineering, and mathematics. However, as is well known, orthogonal wavelet bases are constrained and are not optimal for some applications. For this and other reasons, semi-orthogonal, bi-orthogonal, and multi-wavelet bases have been introduced and studied. The paper by Han, Kwon, and Park falls in the category of multi-wavelets. This paper provides an effective algorithm for the construction of compactly supported Riesz and semi-orthogonal multi-wavelet bases, starting from two-scale vector equations, with illustrative examples of short matrix-valued filters.

It is well known that the wavelet decomposition of a function can provide information about the regularity of the function, as well as its local and global behavior, as long as the analysis wavelet that underlies the decomposition is regular, is smooth, and has sufficient vanishing moments. In other words, the size of the wavelet coefficients provides sufficient information about the membership, in certain functional spaces, of which the function represented by these coefficients is supposed to be. The paper by Führ and Wild describes an analogous connection between the decay of the discrete wavelet transform of a sequence and the membership of this sequence in the discrete Besov spaces introduced by Torres. In spite of this seemingly intimate analogy, the results in this paper do not directly follow from the continuous-time theory and are well adapted to practical applications.

Affine wavelets with a desirably high order of vanishing moments provide a powerful tool for the analysis of point "singularities." For more effective analysis of higher dimensional singularities, such as multi-scale edges in a digital image, the notions of curvelets, contourlets, beamlets, and so forth were introduced. In order to maintain the affine-typed structure of wavelets for higher dimensional singularity analysis, the paper by Guo, Labate, Lim, Weiss, and Wilson introduces an interesting idea to generalize the dilation (expansive) matrix operation by tacking on an additional multilevel multiplicative matrix operation which is not necessarily expansive. Positive results, including assurance of the existence of such multi-wavelets for a general class of pairs of expansive and non-expansive matrices, are established. A very general theory is developed and the study is quite extensive. In addition, several open problems are formulated to facilitate further advancement of this new multivariate multi-wavelet approach.

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While the introduction of wavelets to the engineering community less than two decades ago contributed to stimulating the recent advancement in time–frequency analysis, the gap between theoretical development and practical applications has significantly narrowed, and will continue to narrow, with increasing demands for more powerful and specific time–frequency processing tools. Mobile communication is perhaps the most important area that requires such technological break-through. The paper by Strohmer that studies linear time-varying operators arising in mobile communication from time–frequency analysis provides an innovative point of view for this application. It is shown that a wireless transmission channel can be modeled as a pseudodifferential operator, and that Gabor bases can be used to approximately diagonalize such operators, resulting in associated matrices that belong to certain Wiener-type Banach algebra with exponentially fast off-diagonal decay. The results in this paper are then used to construct numerically efficient equalizers for multi-carrier communication systems in a mobile environment.

Sampling and reconstruction problems in shift-invariant spaces bring together ideas from functional analysis, frame theory, wavelet analysis, and approximation theory. They have important applications at a time when digital communication and processing are prevalent. The paper by Aldroubi and Krishtal describes the problem of sampling in shift-invariant spaces. The results show that a set of sampling in a certain space from which reconstruction is possible remains a set of sampling even if the space is perturbed or, alternatively, if the sampling functional is altered by perturbation. An extension of the results of Beurling and Landau on the density of sampling sets follows from the results in this paper.

Ever since the early development of wavelet theory, the class of polynomial B-splines has played a central roll, either directly or indirectly, in the introduction and formulation of multi-resolution spaces, which in turn provide the architecture for the construction of the most important and useful wavelets. The paper by Forster, Blu, and Unser introduces the notion of complex B-splines, by extending analytically the integer and fractional degrees of the Fourier transform of cardinal B-splines and fractional B-splines, respectively, to complex-valued degrees. They show that these new splines retain most of the properties of cardinal polynomial B-splines, including smoothness and two-scale relations, while offering certain degrees of freedom with respect to time and frequency shifting. In particular, it is proved that the complex B-splines are asymptotically equivalent to certain (phase) modulated Gaussians and that they tend to be optimally localized in the sense of the Heisenberg uncertainty principle.

The theory of frames introduced by Duffin and Schaeffer over 50 years ago as an abstract generalization of non-harmonic Fourier series is now an active area of mathematical research and has many important consequences, even in applications such as communication theory. The paper by Tang and Weber discusses the important class of harmonic or geometrically uniform frames, and more generally, Bessel sequences. These frames arise from the action of an Abelian group of unitary operators. It is shown, in particular, that these frames are characterized by certain "local" properties in finite dimensional spaces.

An initial step in the investigation of frames is the study of Bessel sequences, while the usually more difficult problem of studying lower frame bounds requires a different approach, such as duality consideration. The paper by Jia derives a set of sufficient conditions for the identification of non-uniform multilevel compactly supported multivariate Bessel sequences, not only for the usual L^2 space, but also for the Sobolev and Besov spaces. The setting considered in this paper is quite general, covering a large class of irregular wavelet-like families and representing a general formulation of irregular "vaguelettes." Furthermore, the set of sufficient conditions introduced in this paper is very weak and, in some sense, close to being optimal.

We thank the authors for submitting their work for publication in this SICHA, and the referees for their conscientious reviews that help maintain the high quality of this journal.

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